Assignment 1

1. Sentence formation in Sentential Logic

By repeated application of the sentence-forming operators ~,&,v, and $\rightarrow$, it is possible to form sentences as complicated as may be needed. For example, since P, Q, R, and S are sentence letters, we can form sentences such as $P\rightarrow{\sim}Q$, ${\sim}P \rightarrow Q$, ${\sim}(P\rightarrow{\sim}Q)$, $P\rightarrow({\sim}Q\rightarrow R)$, $P\rightarrow{\sim}(Q\rightarrow R)$, $(P\rightarrow Q)\rightarrow(R\rightarrow S)$.

In these examples, I have used sentence letters, connectives and parentheses. The purpose of the parentheses is to indicate how a complex expression is constructed from its parts. So, the use of parentheses in logical notation is exactly the same as the use of parentheses in elementary mathematical notation. In 6+(5x4), we first multiply 5 and 4 and then add 6 to the result, whereas in (6+5)x4, we first add 6 and 5 and then multiply the result by 4. The order makes a difference; the results are different. Analogously, in ${\sim}P\rightarrow Q$, we first form the negation of P and then form the conditional whose antecedent is ~P and whose consequent is Q: this process is represented in Tree 1 below. In ${\sim}(P\rightarrow Q)$, however, we first form the conditional $P\rightarrow Q$ from P and Q, and then apply negation to the whole conditional. This process is represented in Tree 2. As in mathematics, the order can make a difference; the sentence ${\sim}P\rightarrow Q$ is not equivalent in logic to the sentence ${\sim}(P\rightarrow Q)$. There is a similar difference in arrangement between $P\rightarrow(Q\rightarrow R)$ and $(P\rightarrow Q)\rightarrow R$, as Trees 3 and 4 show.

Tree 1 Tree 2 Tree 3 Tree 4

Understanding how differences in placement of propositional letters and parentheses can affect the logical content of an expression is perhaps the most important ability to acquire in thinking about sentential logic. As a guide, here are some examples to think about. In each case, the sentences are not logically equivalent. (You should return to these examples later to check your understanding of them.)

1. $P \rightarrow Q$ $Q \rightarrow P$	2. ${\sim}(P\&Q)$ ${\sim}P \& Q$	3. ${\sim}(PvQ)$ ${\sim}P$ v Q	4. $P\rightarrow(Q\rightarrow R)$ $(P\rightarrow Q)\rightarrow R$
5. $P\rightarrow(Q\&R)$ $(P\rightarrow Q)\&R$	6. $(P\&Q) \rightarrow R$ $P\&(Q\rightarrow R)$	7. P&(QvR) (P&Q)vR	8. $(PvQ)\rightarrow R$ $Pv(Q\rightarrow R)$

The diagrams used on Page 1 are called construction trees. These trees show (from bottom to top) how a sentence is built up from its parts. Because of the way in which parentheses are used, each sentence of sentential logic has only one correct construction tree.

How do you determine the correct construction tree for a sentence of sentential logic? For example, how is the sentence:

$\sim((P\to(Q\to R))\to(\sim(P\to\sim R)\to S))$

constructed? One simple strategy for answering this question is to pair up parentheses from the inside out. First, pair up those left and right parentheses which have no intervening parentheses, thus:

$\sim((P\to(Q\to R))\to(\sim(P\to\sim R)\to S))$

This shows what elements are put together first, at the bottom of the construction tree. Now, join the next closest left-right pairs. This yields:

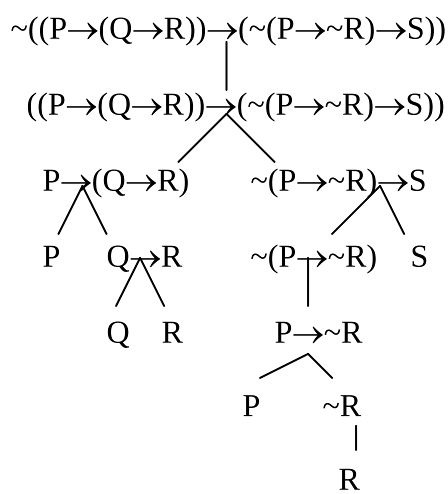
$\sim((P\to(Q\to R))\to(\sim(P\to\sim R)\to S))$

This shows what elements are to be grouped together at the next highest level. Proceed in this way until all parentheses are connected:

$\sim((P\to(Q\to R))\to(\sim(P\to\sim R)\to S))$

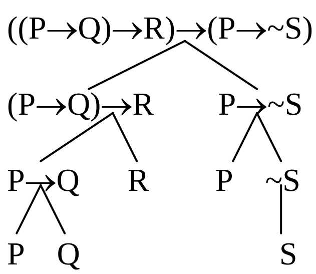
We can then produce a construction tree using this parenthesis pairing as a guide:

Tree 5

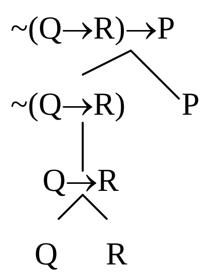


Here are a few more examples.

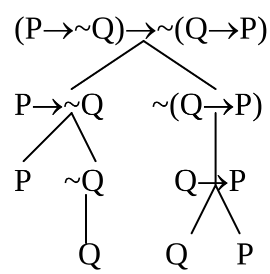
Tree 6



Tree 7



Tree 8



The <u>main connective</u> in a sentence is the <u>top</u> connective introduced in the construction tree. In Tree 1, the main connective is the arrow; in Tree 2, it is the negation sign; in Tree 3, the <u>first</u> arrow, and in Tree 4, the <u>second</u> arrow. What is the main connective in trees 5, 6, 7, 8? It is going to be important for using the rules introduced shortly to correctly identify the main connective in each sentence. It is important because the rules can be correctly applied <u>only</u> to the main connective in a sentence.

2. Paraphrasing English into Sentential Logic

In sentential logic, '~' is to be read 'it is not the case' and '→' is to be read 'if….then____'. However, if we want to apply sentential logic to the analysis of arguments stated in English, we need to be sensitive to the variety of ways in which negation and conditionality can be expressed in English.

Negation can be expressed by an initial prefix 'it isn't the case', an internal 'not', or by contraction 'n't'. Thus, we can express the negation of 'Jack climbed the hill' by 'It is not the case that Jack climbed the hill', 'Jack did not climb the hill', or 'Jack didn't climb the hill'. Sometimes, the prefix 'un' or 'in' is used to express the negation of a sentence, but not always. For example, 'The table is unpainted' is synonymous with the negation of 'The table is painted'. However, 'Tom is unhappy' is not, in my opinion, synonymous with the negation of 'Tom is happy'.

Conditionality may be expressed in English in a variety of ways. We can express "If A, then B" by any of the following: 'provided A, B', 'assuming A, B', 'given that A, B', 'B on the condition that A'. Another important conditional idiom is 'A only if B'. To see what this means, note that the sentence 'It will snow only if the temperature is below 40 degrees is a true sentence. What this means is that if the temperature is <u>not</u> below 40 degrees, then it will <u>not</u> snow; symbolically, this is ~B→~A, where B is 'The temperature is below 40 degrees and A is 'It will not snow'. It <u>doesn't</u> mean that if the temperature is below 40 degrees then it will snow. The connective 'unless' also expresses conditionality; how would you express it symbolically? Think about this example; it won't snow unless the temperature is below 40 degrees.

The connectives '&' and 'v' are to be read as 'and' and 'or' respectively. Conjunction (&) can be expressed in English in a variety of ways, including 'A and B', 'A, but B', 'A; moreover, B', 'A; nevertheless B', 'A; however, B'. Disjunction (v) is typically expressed in the form 'A or B'. There are other English locutions which can be expressed using negation, conjunction and disjunction. Two important examples are 'Neither A nor B' and 'Not both A and B'. How should these forms be represented symbolically?

If you are paraphrasing a sentence which contains several connectives, it is important that the translation order these connectives correctly; compare these sentences:

a. It is not the case that if Joel quits his job, then he'll be happy.

b. If Joel does not quit his job, then he'll be happy.

In sentence a, negation applies to the whole conditional, e.g. ~(P→Q), whereas in sentence b, it applies only to the antecedent, e.g. ~P→Q.